Numbers We Can Believe in?
— Part 1: Counterfactual and Welfare Analysis —

This Lecture

1. The Simplest Gravity Model: Armington
2. Gravity Models and the Gains from Trade: ACR (AER 2012)
3. Beyond ACR's Equivalence Result: CR (Handbook 2014)

1. The Simplest Gravity Model:

Armington

The Armington Model

Overview of the Armington Model

- Many countries indexed by $i = 1, ..., n$
- Each country is endowed with L_i units of labor
- Each country can produce one good one-to-one from labor
  - Trade between country i and j is subject to iceberg trade costs $\tau_{ij} \geq 1$
- Each country has a representative agent with CES utility
  - $\sigma =$ elasticity substitution between goods from different countries

Equilibrium of Armington Model

- In equilibrium: consumers maximize utility + labor market clear
- Utility maximization $\Rightarrow$ bilateral trade flows satisfy **gravity equation**:

$$X_{ij} = \frac{\left(w_i \tau_{ij}\right)^{1-\sigma}}{\sum_{l=1}^{n} \left(w_l \tau_{lj}\right)^{1-\sigma}} w_j L_j$$

- Labor market clearing $\Rightarrow$ wages $\{w_i\}$ solve non-linear system:

$$w_i L_i = \sum_j \frac{\left(w_i \tau_{ij}\right)^{1-\sigma}}{\sum_{l=1}^{n} \left(w_l \tau_{lj}\right)^{1-\sigma}} w_j L_j$$

- In what follows $\varepsilon \equiv -\frac{d \ln X_{ij}/X_{jj}}{d \ln \tau_{ij}} = \sigma - 1$ denotes the **trade elasticity**

Counterfactual and Welfare Analysis

- **Question:**
  Consider a foreign shock: $L_i \to L_i'$ for $i \neq j$ and $\tau_{ij} \to \tau_{ij}'$ for $i \neq j$.
  How do foreign shocks affect real consumption, $C_j \equiv w_j / P_j$?

- Let P_j denote the CES price index. Shephard's Lemma implies

$$d \ln C_j = d \ln w_j - d \ln P_j = -\sum_{i=1}^{n} \lambda_{ij} \left(d \ln p_{ij} - d \ln p_{jj} \right)$$

  with $p_{ij} \equiv w_i \tau_{ij}$ and $\lambda_{ij} \equiv X_{ij} / w_j L_j$.

- Gravity implies

$$d \ln \lambda_{ij} - d \ln \lambda_{jj} = -\varepsilon \left(d \ln p_{ij} - d \ln p_{jj} \right).$$

Counterfactual and Welfare Analysis

- Combining these two equations yields

$$d \ln C_j = \frac{\sum_{i=1}^{n} \lambda_{ij} \left(d \ln \lambda_{ij} - d \ln \lambda_{jj} \right)}{\varepsilon}.$$

- Noting that $\sum_i \lambda_{ij} = 1 \Longrightarrow \sum_i \lambda_{ij} d \ln \lambda_{ij} = 0$ then

$$d \ln C_j = -\frac{d \ln \lambda_{jj}}{\varepsilon}.$$

- Integrating the previous expression yields $(\hat{x} = x'/x)$

$$\hat{C}_j = \hat{\lambda}_{jj}^{-1/\varepsilon}.$$

So What is the Welfare Impact of a Foreign Shock?

- In general, predicting $\hat{\lambda}_{jj}$ requires (computer) work
  - We can use exact hat algebra as in Dekle, Eaton and Kortum (2008)
  - Using gravity equation + data $\{\lambda_{ij}, Y_j\}$, and ε we can solve for counterfactual changes in wages (up to choice of numeraire)

$$\widehat{w}_i = \sum_{j=1}^{n} \frac{\lambda_{ij}\hat{w}_j Y_j \left(\hat{w}_i\hat{\tau}_{ij}\right)^{\varepsilon}}{Y_i \sum_{i'=1}^{n} \lambda_{i'j}\left(\hat{w}_{i'}\hat{\tau}_{i'j}\right)^{\varepsilon}}.$$

  - Then we can compute change in sufficient statistic for welfare:

$$\widehat{\lambda}_{jj} = \left[\sum_{i=1}^{n} \lambda_{ij}\left(\hat{w}_i\hat{\tau}_{ij}\right)^{\varepsilon}\right]^{-1},$$

- Predicting how bad it would be to shut down (all) trade is easier...
  - In autarky, $\lambda_{jj} = 1$. So

$$C_j^A / C_j = \lambda_{jj}^{1/\varepsilon}$$

  - Thus **gains from trade** can be computed as

$$GT_j \equiv 1 - C_j^A / C_j = 1 - \lambda_{jj}^{1/\varepsilon}$$

Gains from Trade According to the Armington Model

- Suppose that we have estimated trade elasticity using gravity equation
  - Central estimate is $\varepsilon = 5$; see Head and Mayer (Handbook, 2014)
- Using World Input Output Database (2008) to get λ_{jj}, we get GT_j:

	λ_{jj}	% GT_j
Canada	0.82	3.8
Denmark	0.74	5.8
France	0.86	3.0
Portugal	0.80	4.4
Slovakia	0.66	7.6
U.S.	0.91	1.8

Cheese, really?

2. Gravity Models and the Gains from Trade: ACR (AER, 2012)

ACR's Main Equivalence Result

- ACR focus on gravity models
  - PC: Armington and Eaton & Kortum '02
  - MC: Krugman '80 and many variations of Melitz '03
- Within that class, welfare changes are ($\hat{x} = x'/x$)

$$\hat{C} = \hat{\lambda}^{1/\varepsilon}$$

- **Two sufficient statistics** for welfare analysis are:
  - Share of domestic expenditure, λ;
  - Trade elasticity, ε
- **Two views** on ACR's result:
  - Pessimistic: within that class of models, micro-level data do not matter
  - Optimistic: welfare predictions of Armington model are more robust/credible than you thought

Primitive Assumptions

Preferences and Endowments

- **CES utility**
  - Consumer price index,

$$P_i^{1-\sigma} = \int_{\omega \in \Omega} p_i(\omega)^{1-\sigma} d\omega,$$

- **One factor of production:** labor
  - $L_i \equiv$ labor endowment in country i
  - $w_i \equiv$ wage in country i

Primitive Assumptions
Technology

- **Linear cost function:**

$$C_{ij}(\omega, t, q) = \underbrace{q w_i \tau_{ij} \alpha_{ij}(\omega)\, t^{\frac{1}{1-\sigma}}}_{\text{variable cost}} + \underbrace{w_i^{1-\beta} w_j^{\beta} \xi_{ij} \phi_{ij}(\omega)\, m_{ij}(t)}_{\text{fixed cost}},$$

q : quantity,
τ_{ij} : iceberg transportation cost,
$\alpha_{ij}(\omega)$: good-specific heterogeneity in variable costs,
ξ_{ij} : fixed cost parameter,
$\phi_{ij}(\omega)$: good-specific heterogeneity in fixed costs.

Primitive Assumptions

Technology

- **Linear cost function:**

$$C_{ij}(\omega, t, q) = qw_i\tau_{ij}\alpha_{ij}(\omega)\, t^{\frac{1}{1-\sigma}} + w_i^{1-\beta} w_j^{\beta} \xi_{ij}\phi_{ij}(\omega)\, m_{ij}(t)$$

$m_{ij}(t)$: cost for endogenous destination specific technology choice, t,

$$t \in [\underline{t}, \overline{t}]\ ,\ \ m'_{ij} > 0,\ \ m''_{ij} \geq 0$$

Primitive Assumptions

Technology

- **Linear cost function:**

$$C_{ij}(\omega, t, q) = qw_i\tau_{ij}\alpha_{ij}(\omega)\, t^{\frac{1}{1-\sigma}} + w_i^{1-\beta} w_j^{\beta} \xi_{ij}\phi_{ij}(\omega)\, m_{ij}(t)$$

- Heterogeneity across goods

$$G_j(\alpha_1, ..., \alpha_n, \phi_1, ..., \phi_n) \equiv \{\omega \in \Omega \mid \alpha_{ij}(\omega) \leq \alpha_i,\ \phi_{ij}(\omega) \leq \phi_i,\ \forall i\}$$

Primitive Assumptions

Market Structure

- **Perfect competition**
  - Firms can produce any good.
  - No fixed exporting costs.
- **Monopolistic competition**
  - Either firms in i can pay $w_i F_i$ for monopoly power over a random good.
  - Or exogenous measure of firms, $\overline{N}_i < \overline{N}$, receive monopoly power.
- Let N_i be the measure of goods that can be produced in i
  - Perfect competition: $N_i = \overline{N}$
  - Monopolistic competition: $N_i < \overline{N}$

Macro-Level Restrictions

Trade is Balanced

- Bilateral trade flows are

$$X_{ij} = \int_{\omega \in \Omega_{ij} \subset \Omega} x_{ij}(\omega)\, d\omega$$

- **R1** *For any country* j,

$$\sum_{i \neq j} X_{ij} = \sum_{i \neq j} X_{ji}$$

  - Trivial if perfect competition or $\beta = 0$.
  - Non trivial if $\beta > 0$.

Macro-Level Restrictions

Profit Share is Constant

- **R2** *For any country* j,

$$\Pi_j / \left(\sum_{i=1}^{n} X_{ji} \right) \text{ is constant}$$

where Π_j : aggregate profits gross of entry costs, $w_j F_j$, (if any)

- Trivial under perfect competition.
- Direct from Dixit-Stiglitz preferences in Krugman (1980).
- Non-trivial in more general environments.

Macro-Level Restriction

CES Import Demand System

- *Import demand system*

$$(\mathbf{w}, \mathbf{N}, \boldsymbol{\tau}) \rightarrow \mathbf{X}$$

- **R3**

$$\varepsilon_j^{ii'} \equiv \partial \ln \left(X_{ij} / X_{jj} \right) / \partial \ln \tau_{i'j} = \begin{cases} \varepsilon < 0 & i = i' \neq j \\ 0 & otherwise \end{cases}$$

- Note: symmetry and separability.

- The *trade elasticity* ε is an *upper-level* elasticity: it combines
  - $x_{ij}(\omega)$ (*intensive margin*)
  - Ω_{ij} (*extensive margin*).
- R3 $\implies$ complete specialization.
- R1-R3 are not necessarily independent
  - If $\beta = 0$ then R3 $\implies$ R2.

Macro-Level Restriction

Strong CES Import Demand System (AKA Gravity)

- **R3'** The IDS satisfies

$$X_{ij} = \frac{\chi_{ij} \cdot M_i \cdot \left(w_i \tau_{ij}\right)^{\varepsilon} \cdot Y_j}{\sum_{i'=1}^{n} \chi_{i'j} \cdot M_{i'} \cdot \left(w_{i'} \tau_{i'j}\right)^{\varepsilon}}$$

  where χ_{ij} is independent of $(\mathbf{w}, \mathbf{M}, \boldsymbol{\tau})$.

- Same restriction on $\varepsilon_j^{ii'}$ as R3 but, but additional structural relationships

Welfare results

- State of the world economy:

$$\mathbf{Z} \equiv (\mathbf{L}, \boldsymbol{\tau}, \boldsymbol{\xi})$$

- *Foreign shocks*: a change from $\mathbf{Z}$ to $\mathbf{Z}'$ with no domestic change.

Equivalence (I)

- **Proposition 1:** *Suppose that R1-R3 hold. Then*

$$\widehat{W}_j = \widehat{\lambda}_{jj}^{1/\varepsilon}.$$

- Implication: 2 sufficient statistics for welfare analysis $\widehat{\lambda}_{jj}$ and ε
- New margins affect structural interpretation of ε
    - ...and composition of gains from trade (GT)...
    - ... but size of GT is the same.

Gains from Trade Revisited

- Proposition 1 is an *ex-post* result... a simple *ex-ante* result:
- **Corollary 1:** *Suppose that R1-R3 hold. Then*

$$\widehat{W}_j^A = \lambda_{jj}^{-1/\varepsilon}.$$

Equivalence (II)

- A stronger ex-ante result for **variable trade costs** under R1-**R3'**:
- **Proposition 2:** *Suppose that R1-R3' hold. Then*

$$\widehat{W}_j = \widehat{\lambda}_{jj}^{1/\varepsilon}$$

where

$$\widehat{\lambda}_{jj} = \left[\sum_{i=1}^{n} \lambda_{ij} \left(\hat{w}_i \hat{\tau}_{ij}\right)^{\varepsilon}\right]^{-1},$$

and

$$\widehat{w}_i = \sum_{j=1}^{n} \frac{\lambda_{ij} \hat{w}_j Y_j \left(\hat{w}_i \hat{\tau}_{ij}\right)^{\varepsilon}}{Y_i \sum_{i'=1}^{n} \lambda_{i'j} \left(\hat{w}_{i'} \hat{\tau}_{i'j}\right)^{\varepsilon}}.$$

- ε and $\{\lambda_{ij}\}$ are sufficient to predict $\widehat{W}_j$ (ex-ante) from $\hat{\tau}_{ij}$, $i \neq j$.

Taking Stock

- ACR consider models featuring:
  - (i) Dixit-Stiglitz preferences;
  - (ii) one factor of production;
  - (iii) linear cost functions; and
  - (iv) perfect or monopolistic competition;

  with three macro-level restrictions:
  - (i) trade is balanced;
  - (ii) aggregate profits are a constant share of aggregate revenues; and
  - (iii) a CES import demand system.
- Equivalence for ex-post welfare changes and GT
  - under R3' equivalence carries to ex-ante welfare changes

3. Beyond ACR's Equivalence Result: CR (Handbook, 2014)

Beyond ACR's Equivalence Result

1. Add multiple sectors
   - Typically nested CES preferences, with different elasticities of substitution between and within sectors
   - Here Cobb-Douglas between sectors
2. Add traded intermediates
   - Typically nested CES technologies, with different elasticities of substitution between and within different types of inputs
   - Here Cobb-Douglas between intermediates
3. Add imperfect competition
   - Typically monopolistic competition
   - Here with and without firm heterogeneity

Gains from Trade

....	Canada	China	Germany	Romania	US
Aggregate	**3.8**	**0.8**	**4.5**	**4.5**	**1.8**

Gains from Trade Redux

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Aggregate	3.8	0.8	4.5	4.5	1.8
MS, PC	**17.4**	**4.0**	**12.7**	**17.7**	**4.4**

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MS, MC	**15.3**	**4.0**	**17.6**	**12.7**	**3.8**

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MS, IO, PC	**29.5**	**11.2**	**22.5**	**29.2**	**8.0**

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MS, IO, MC (Krugman)	**33.0**	**28.0**	**41.4**	**20.8**	**8.6**

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MS, IO, MC (Krugman)	33.0	28.0	41.4	20.8	8.6
MS, IO, MC (Melitz)	**39.8**	**77.9**	**52.9**	**20.7**	**10.3**

From Gains from Trade to Trade Policy Evaluation

- In Armington: back to $\{\lambda_{ij}, Y_j\}$, ε and $\left\{\hat{tariff}_{ij}\right\}$ to get implied $\hat{\lambda}_{jj}$

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  - Use exact algebra (ACDshow how to do this nonparametrically)
  - Just need more elasticities (Preferences, IO linkages etc.)
- By construction: calibrated model always exactly matches data!
- **Question**: Does that make counterfactual predictions "credible"?

Still a pretty restrictive class of models...